Repetition Pitch glide from the step pyramid at Chichen Itza (L)

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Standing at the foot of the Mayan step pyramid at Chichen Itza in Mexico, one can produce a pitchy "chirp" echo by handclapping. As exposed by Declercq *et al.* [J. Acoust. Soc. Am. **116**, 3328–3335 (2004)], an acoustic model based on optical Bragg diffraction at a periodic structure cannot explain satisfactorily the chirp-echo sonogram. Alternatively, considering the echo as a sequence of reflections, and given the dimensions of the pyramid and source-receiver position, the chirp is predicted correctly as a Repetition Pitch glide of which the pitch height is continuously decreasing within 177 ms from 796 to 471 Hz-equivalent. © *2006 Acoustical Society of America.*
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I. INTRODUCTION

At Chichen Itza in Mexico there is a Mayan ruin with a pyramid named El Castillo that produces an echo, in response to a handclap, which sounds like the chirp of a Quetzal bird. It was correctly suggested by Lubman[1] and others that the periodic structure of, in particular, the central staircase is responsible for the chirp-like sound of the echo. In search for an explanation, Declercq *et al.*[2] performed sonogram analysis of the echo as recorded by Lubman,[1] and applied the monofrequent single homogeneous plane wave (Rayleigh) diffraction theory to the periodic structure of the steps of the pyramid and the handclap pulse under consideration.

In Fig. 1, the gray-scaled background constitutes the sonogram of the chirp echo as reproduced from Fig. 8 of Declercq *et al.*;[2] they applied a time-limited Fourier transform with a Gaussian window of 2 ms width to the chirp echo as recorded by Lubman.[1] Time is plotted horizontally on a linear scale from 0 to 200 ms, and frequency vertically on a linear scale from 0 to 5000 Hz. In the text below their Fig. 8, they state that "it can already be concluded that these patterns cannot simply be the result of pure Bragg diffraction and that an extra effect must be involved."[3]

It occurred to the present author, within the framework of the Repetition Pitch theory, that the lighter areas in this sonogram suggest the presence of spectral energy at the second, third, and fourth harmonic of a gliding (absent) fundamental frequency. The other lighter regions immediately above 0 Hz are ascribed by Declerq *et al.*[2] to low-frequency noise coming from the interaction of wind with the microphone. Therefore, an alternative explanation based on the perception of Repetition Pitch is proposed in this letter.

II. AN ALTERNATIVE MODEL BASED ON REPETITION PITCH

When a sound and its (delayed) repetition(s) are added together, a compound signal is obtained having a rippled spectrum. In particular, the power spectrum of a white signal with one added repetition is a cosinusoidal function of frequency with spectral maxima at multiples of a "fundamental" corresponding to the reciprocal value of the delay time. With such a signal, one generally perceives a pitch, Repetition Pitch (RP), corresponding to the fundamental.[4,5] This is true for different kinds of RP signals ranging from pulse pairs to continuous white noise added to itself delayed.[6–8] In the present case, the original sound is a handclap, to be idealized as a (dirac) pulse. Paired reflections (repetitions) from successive steps will be considered as pulse pairs of which a (gliding) fundamental together with its harmonics is calculated.

Adopting the data of pyramid dimensions, handclap and sound recording positions by Lubman from Declercq *et al.*,[2] the drawing in Fig. 2 is obtained. The steps of the staircase are numbered $n=-7$ to 84, with $n=0$ being the step at ear (microphone) height. Dimensions are given in meters.

The sound path length $S(n)$ from source-receiver position to step n of the staircase is calculated with Pythagoras' theorem as follows:

$$S(n) = [(11.8 + n \times 0.263)^2 + (n \times 0.263)^2]^{1/2} \quad \text{with } n = -7, \ldots, 0, \ldots, 84. \tag{1}$$

Then, with a sound velocity at the site of 343 m/s, the traveling time back and forth, $T(n)$ (in seconds), follows as

$$T(n) = 2 \times S(n)/343 \quad \text{with } n = -7, \ldots, 0, \ldots, 84. \tag{2}$$

The time interval or delay time $\tau(n)$ between successive reflections at the receiver position then is given by

$$\tau(n) = T(n+1) - T(n) \quad \text{with } n = -7, \ldots, 0, \ldots, 84. \tag{3}$$

Finally, the instantaneous value of the "fundamental" follows as the reciprocal value of $\tau(n)$ and the "harmonics" f_m of the fundamental are given by

$$f_m(n) = m/\tau(n) \quad \text{with } m = 1, 2, 3, 4, \ldots \, . \tag{4}$$

The course (glide) of the first four harmonics is given in Fig. 1 by the dotted lines numbered 1 through 4, with each dot representing the instantaneous value of a RP harmonic following Eq. (4). The calculated fundamental glides from 796 to 471 Hz within a time span of 177 ms, the latter value

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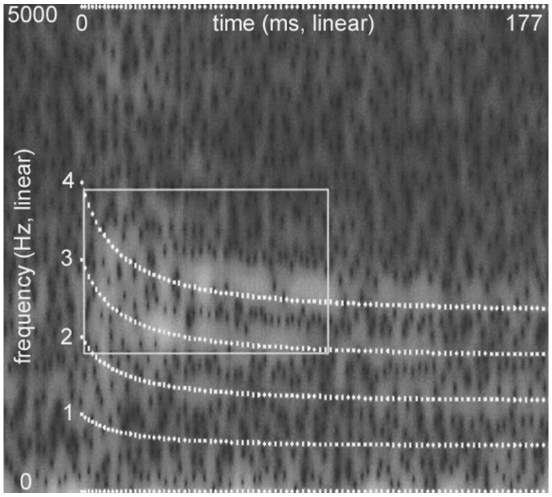

FIG. 1. Sonogram produced by Declercq *et al.* (Ref. 2, their Fig. 8) of the isolated chirp echo as recorded by Lubman (Ref. 1). Time-limited Fourier transformation with a Gaussian window of 2 ms was applied. Vertical axis: frequency linear from 0 to 5000 Hz; horizontal axis: time linear from 0 to 200 ms; white indicates high intensity. The white rectangle has no specific meaning for the present considerations. Dotted lines marked 1 through 4 constitute the present model predictions (see the text).

being the sum of all individual delays. For easy comparison with the sonogram of the recorded chirp echo, the dotted lines are superimposed on the sonogram (total span 200 ms) so that the 177-ms span of the model (91 dots) coincides as well as possible with the extent of the lighter areas of the sonogram in the horizontal direction (three dots fall outside the figure). Horizontal dotted lines at 0 Hz and 5 kHz coincide with the frequency scale of the sonogram. It can be concluded that the dotted lines fit nicely to the lighter regions in the sonogram.

III. CONCLUDING REMARKS

In order to correctly appreciate the fit between the sonogram and the theoretical curves, one has to take into account that the 2-ms Gaussian time window of the sonogram's running Fourier transform (see Declercq *et al.*[2]) covered mainly two reflections, given a delay time interval of 1.95 ms (=177/91) on average. This implies that, in accordance with our calculations based on pulse pairs, the vertical width of the lighter areas is expected to reflect cosinusoidal power spectra.[9]

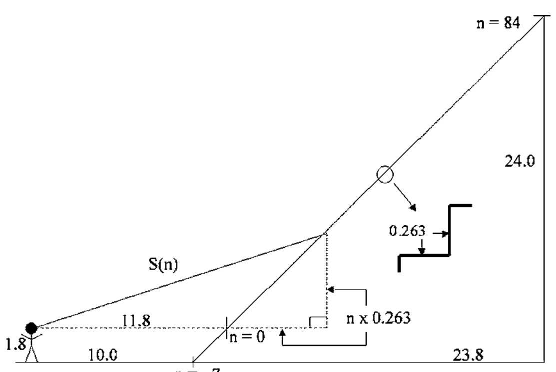

FIG. 2. Schematics of El Castillo pyramid in Chichen Itza following the data by Lubman as adopted by Declercq *et al.* (Ref. 2). Dimensions are given in meters. Source-receiver position is at a height of 1.8 m at a distance of 10 m from the foot of the pyramid. Step width is 0.263 m. Steps are numbered $n=-7$ to 84.

As far as perception is concerned, it should be noticed that no spectral energy is present in the signal at the "fundamental." Thus, a Repetition Pitch is predicted due to the presence of the second and, more important, the third and fourth "harmonics" (the so-called dominant harmonics[5,6]). It glides from 796 to 471 Hz-equivalent. That is to say: in theory, because we have not considered the short-term characteristics of the cochlear frequency analysis as well as the integration characteristics of the human pitch processor. This would be beyond the limited scope of the present letter. By informal listening, instead, to a synthesized RP glide following the above-noted model, great similarity with the chirp recorded by Lubman[1] (downloaded from his website) was observed. This confirms also the perceptual relevance of the present considerations.

Additionally, it is worthwhile mentioning an original observation by Christian Huygens[10] in 1693 at the castle of Chantilly de la Cour in France. Standing at the foot of the majestic stone staircase in the garden he noticed that the noisy sound from a fountain produced a certain pitch. This was confirmed at later occasions by handclapping. Acoustic registrations show a regular reflection pattern due to the equality in width of the steps of the staircase, resulting in a stationary Repetition Pitch of 370 Hz-equivalent. Thus, both continuous (fountain) noise and handclap are understood to be appropriate source signals in Chantilly. In this context, it is tempting to predict that a setting with continuous noise would most probably not work at Chichen Itza, because of the simultaneous mixing of all the different time delays in the chirp echo.

ACKNOWLEDGMENT

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[1]D. Lubman, http://www.ocasa.org/MayanPyramid.htm (site visited on 24-11-2005).

[2]N. F. Declercq, J. Degrieck, R. Briers, and O. Leroy, "A theoretical study of special acoustic effects caused by the staircase of the EL Castillo pyramid at the Maya ruins of Chichen Itza in Mexico," J. Acoust. Soc. Am. **116**, 3328–3335 (2004).

[3]Exact, though cumbersome simulations, based on amplitude calculations by means of an extension of the Rayleigh theory of diffraction, produce better results, in agreement with experiments. The theoretical model reveals the amplitude distribution in a sonogram representation of the echo in front of the pyramid. From this amplitude distribution it is also possible to extract the "delay lines" of the echo [N. F. Declercq (private communication)].

[4]F. A. Bilsen, "Repetition Pitch: Monaural interaction of a sound with the repetition of the same, but phase shifted, sound," Acustica **17**, 295–300 (1966).

[5]F. A. Bilsen, "Pitch of noise signals: Evidence for a central spectrum," J. Acoust. Soc. Am. **61**, 150–161 (1977).

[6]F. A. Bilsen and R. J. Ritsma, “Some parameters influencing the perceptibility of pitch,” J. Acoust. Soc. Am. **47**, 469–475 (1970).
[7]W. M. Hartmann, *Signals, Sound, and Sensation* (AIP Press, New York, 1997), pp. 361–376.
[8]T. D. Rossing, *The Science of Sound* (Addison-Wesley, New York, 1990), pp. 121–122.
[9]A prediction of the complete sonogram instead of only spectral maxima (dotted lines) seems feasible partly by incorporating (running) cosinusoidal power spectra of pulse pairs multiplied with the spectrum of the handclap. However, also including reflection properties of the steps (amplitude and phase) is beyond the scope of the present approach.
[10]C. Huygens, “ En envoiant le probleme d’Alhazen en France,” Correspondence No. 2840 (November 1693) printed in Oeuvres Complètes, Vol. **10**, edited by Societý Hollandaise des Sciences (Nijhoff, Den Haag, 1950), pp.570–571.