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## Huygens on pitch perception; staircase reflections reconsidered

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**Abstract**

In 1693, Christian Huygens standing at the foot of the majestic stone staircase in the garden of the castle Chantilly in France noticed that the noisy sound from a fountain produced a certain pitch. He was able to determine the height of this pitch by matching it with the pitch produced by a 'closed organ pipe', made by rolling up a piece of paper, with a length equal to the width of one step of the staircase [Huygens, Oeuvres Complètes **10**, 570-571]. His observations were confirmed in 1993 with modern registration and analysis methods. It is shown in the present paper that the pitch heard by Huygens is of the Repetition Pitch type [Bilsen, e.g. JASA **61** (1977), 150-161] with a height of 370 Hz-equivalent. A similar pitch phenomenon was reported in Mexico [Lubman, www.ocasa.org/MayanPyramid.htm]. Standing at the foot of a step-pyramid, one can produce a pitchy 'chirp' by handclapping. An acoustic model based on optical (Bragg) diffraction at a periodic structure, could not explain the results of sonogram analysis satisfactorily [Declercq et al., JASA **116** (2004), 3328-3335]. On the other hand, if the chirp-echo is considered as a sequence of 90 reflections, and given the dimensions of the pyramid with an handclapping observer at 10 m from its base, it can easily be calculated that the chirp is predicted correctly as a gliding Repetition Pitch of which the pitch height is continuously decreasing within 177 ms from 796 to 471 Hz-equivalent.

# 1. Introduction

When a sound is reflected, often a pitch effect can be heard. Examples are the sound from an airplane flying over and causing a reflection at the soil, or the sound from a noisy sound source reflected against the wall of a house. These cases are typical for a pitch category called Repetition Pitch [1]. In music, the colouring (pitchy) effect of a repetition is sometimes brought about purposely on artistic grounds by using digital delay ('flanging'). If a sound is reflected repeatedly at more or less regular time intervals, this pitch is generally intensified; for example: the metallic sound of handclapping between two parallel walls (flatter echo).

The present paper was triggered by an NAG-presentation and paper by Declercq et al. [2] on a theoretical study of the chirp-like echo sound caused by handclapping at the foot of the staircase of the El Castillo pyramid at the Maya ruins of Chichen-Itza in Mexico. The authors propose an acoustic model based on optical (Bragg) diffraction at a periodic structure. However, their calculations could not explain the results of sonogram analysis satisfactorily. Among other things, it is the purpose of the present paper to offer an alternative explanation based on the perception of a gliding Repetition Pitch.

Therefore, first, a short summary of the main properties of Repetition Pitch will be given. Basically, a sound like white noise presented together with its repetition(s) will produce a spectrum shaped like a comb filter of which spectral maxima ('harmonics') are situated at multiples of a fundamental frequency. Energy at the fundamental may or may not be present in the spectrum. Presented with such a comb spectrum the brain calculates a best-fitting fundamental from the harmonics that are spectrally resolved in the cochlea [3].

Further, Christian Huygens (1693)' original observations and considerations [4] at a big staircase with noisy fountain at the castle of Chantilly in France will be recalled and reconsidered. New measurements with modern recording and analysis methods, together with our present knowledge of Repetition Pitch, will confirm the historic report by Huygens. Finally, a similar reasoning based on Repetition Pitch and the acoustic configuration form the alternative model for a correct prediction of the chirp-echo at the Maya pyramid.

## 2. A short summary of Repetition Pitch

When a sound and its (delayed) repetition are presented together one perceives a pitch, Repetition Pitch (RP), corresponding to the reciprocal value (1/τ) of the delay time (τ) [1]. By applying specific phase shifts to the repetition, pitch shifts are effectuated that formed a clue to alternative pitch theories, be it in the time domain like autocorrelation [1] or in the spectral domain like spectral pattern matching [3]. Adding more repetitions (reflections) with the same delay time generally increases the strength of the pitch sensation. This was re-examined systematically in recent years by a.o. Yost and colleagues with Iterated Rippled Noise (see Hartmann [5] for a review). The (amplitude or power) spectrum of RP-signals generally has a comb or ripple shape with spectral maxima at multiples of the 'fundamental' (1/τ).

Two special cases are known that show a simple closed mathematical expression for the power spectrum. They are shown in figure 1 with their pulse response (h), autocorrelation function (φ) and power spectrum (Φ) respectively; g is the gain of the repetition.

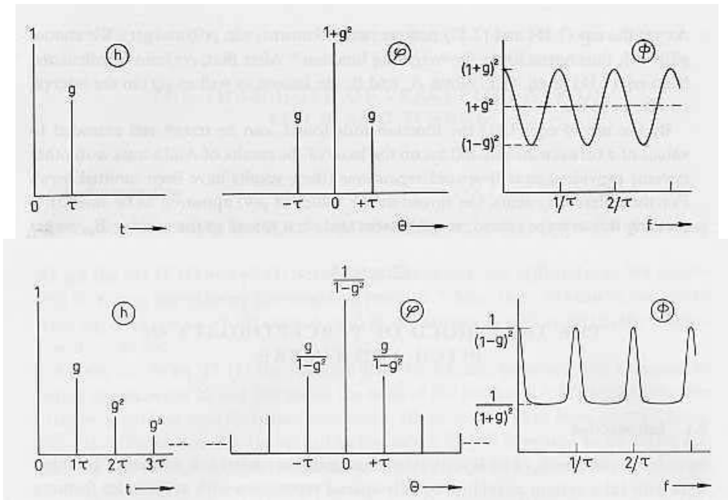


**Figure 1** Two special cases representative for signals with delay time (τ) evoking Repetition Pitch (1/ τ). Pulse response (h), autocorrelation function (φ), and power spectrum (Φ) are shown respectively. Upper panel: one single repetition with gain (g). Lower panel: the signal from a delay line with feedback (compare flatter echo).

Strong arguments for preference of a spectral explanation of Repetition Pitch came from various psychophysical experiments on complex tones generally showing that (a) the lower harmonics are spectrally resolved in the cochlea, and (b) the lower harmonics communicate the strongest musical pitch. These arguments were augmented by electro-physiological experiments with the one-repetition signal [6].

Figure 2 presents the response of a single cell in the cochlear nucleus of cat to a one-repetition signal as a function of τ with fixed parameter g; for details of the experiment see [6]. To get a real picture of the internal spectrum in the cochlear nucleus for one particular value of τ, one has to imagine the response of many different cells (with different characteristic frequencies). Thus, by replacing τ by f (frequency), figure 2 can be imagined to reflect the internal spectrum. It is manifest from this figure that, indeed, the lower harmonics (especially 3rd and 4th) are optimally represented in the auditory system.

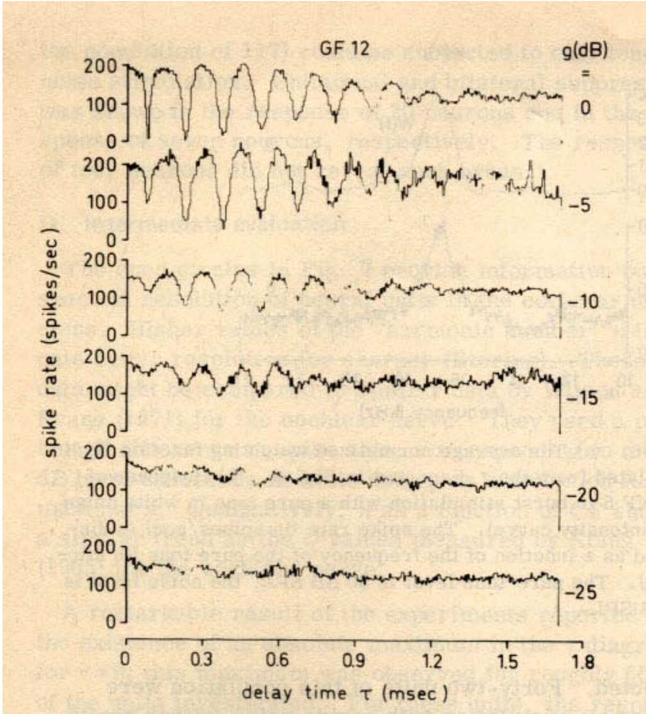


**Figure 2** The response (spikes per second) of a single cell (GF 12) in the cochlear nucleus of cat to white noise with a single repetition, as a function of the delay time τ in ms, with the gain g of the repetition in dB as a fixed parameter (after [6]).

Given this internal spectral representation of signals in the peripheral auditory system, we hypothesize the brain to extract a pitch from the lower (e.g. 3rd, 4th, and 5th) harmonics by calculating a best-fitting fundamental (1st harmonic) [3]. This process of spectral pattern matching is schematised in figure 3 for two cases.

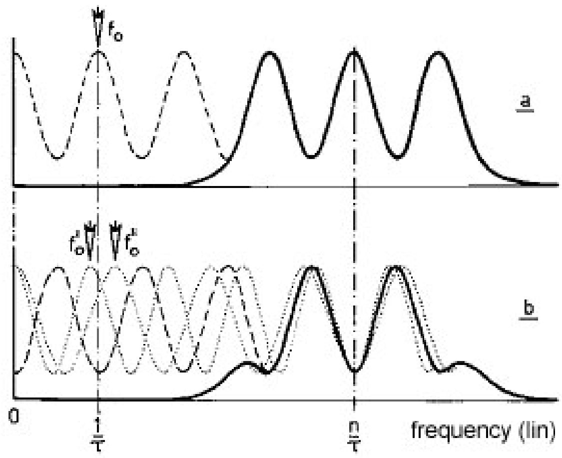


**Figure 3** Extraction of Repetition Pitch by spectral pattern matching. Case (a) represents an harmonic spectrum with a pitch corresponding to $f_0$, case (b) an anharmonic spectrum with two ambiguous pitches (after [3]).

Case (a) represents an harmonic spectrum obtained by appropriate band filtering after adding the repetition (solid line); $f_0$ indicates the exact fitting Repetition Pitch (see dashed line with arrow). Case (b) represents an anharmonic spectrum with two maxima, obtained by band-filtering after subtracting (instead of adding) the repeated sound from the original. In this case there are two alternative best fits, indicated by $f_0$ ‘ and $f_0$ “ respectively. Such a model correctly predicts psychophysical results on ambiguous pitches [1,3].

# 3. A stationary Repetition Pitch at Chantilly

## 3.1 Original observations by Huygens in 1693

The present treatment of acoustic staircase reflections goes back to the famous Dutch scientist Christian Huygens (1629-1695). His inheritance contains a huge number of letters (‘communications’) to colleagues and friends as well as publications on different topics in physics. Well-known are his many contributions to mathematics, astronomy, optics and classical mechanics, as well as his inventions like the clockwork. Less known are his contributions to acoustics, among others treatments on stretched strings, a 31-tone theory on musical temperament and harmony, and several observations in plain air. Apart from being an innovative theoretical and applied scientist, Huygens was a keen observer in daily life, or in Minnaert’s terminology [7]: in physics of the free field.

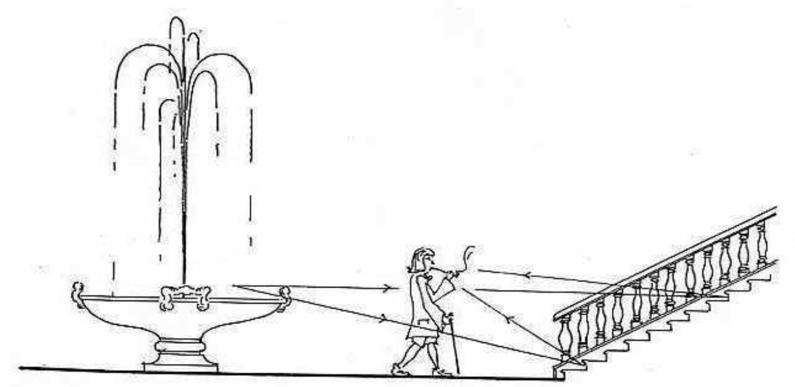

**Figure 4** Christiaan Huygens at Chantilly as imagined by F.M.M. Bilsen (after [1], see also [5])

In 1693, Christian Huygens standing at the foot of the majestic stone staircase in the garden of the castle of Chantilly de la Cour in France noticed that the noisy sound from a fountain produced a certain pitch [4] (see also [7]). The situation is imagined in figure 4, and a photgraph of the staircase is reproduced in figure 5. Huygens presents the following explanation for the pitch perceived (translated rather literally by the present author from old French):

“When one is standing between the staircase and the fountain, one hears from the side of the staircase a resonance that possesses a certain musical pitch that continues, as long as the fountain spouts. One did not know where this tone originated from or improbable explanations were given, which stimulated me to search for a better one. Soon I found that it originated from the reflection of the noise from the fountain against the steps of the staircase. Because like every sound, or rather noise, reiterated in equal small intervals produces a musical tone, and like the length of an organ pipe determines its own pitch by its length because the air pulsations arrive regularly within small time intervals used by the undulations to do the length of the pipe twice in case it is closed at the end, so I imagined that each, even the smallest, noise coming from the fountain, being reflected against the steps of the staircase, must arrive at the ear from each step as much later as the step is remote, and this by time differences just equal to those used by the undulations to travel to and fro the width of one step. Having measured that width equal to 17 inches, I made a roll of paper that had this length, and I found the same pitch that one heard at the foot of the staircase."

**Figure 5** Staircase at the castle of Chantilly de la Cour in France with fountain on the right (not shown)

And having established that the pitch was heard only when the fountain was working, he returned in winter when snow obscured the shape of the steps and he confirmed the absence of the pitch although the fountain was switched on. Really a great experiment: tackling and controlling separately the two factors responsible for the pitch!

Of course, Huygens did not have knowledge of the special properties of Repetition Pitch, but having to do with many regular repetitions instead of only one, he could immediately draw an analogy with the musical tones from organ pipes. Thus, acoustically rather than physiologically, his explanation was complete and final.

## 3.2 Analysis of sound recordings made in 1993

In 1993, video and sound recordings were made at Chantilly with handclapping as basic signal. The audio track (stereo) was analysed with the software program 'GoldWave'. One handclap together with its reflections from the soil, the first 10 steps, and the steps following a plateau are reproduced in figure 6 (compare with figure 5).

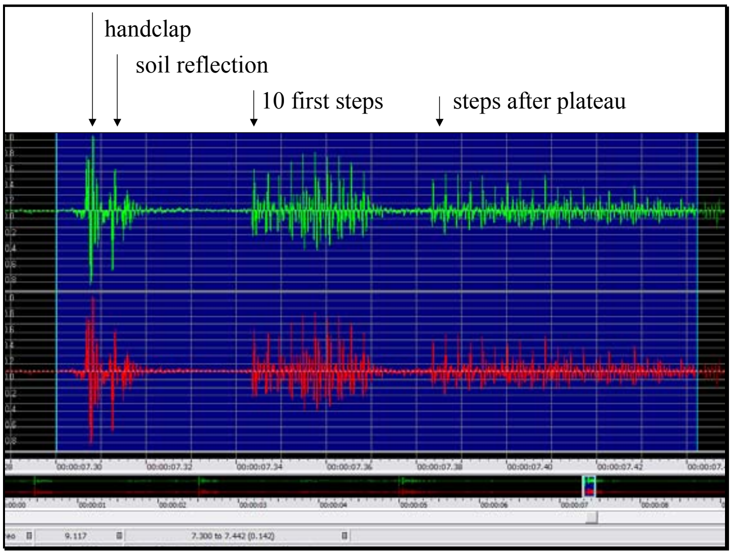


**Figure 6** Sound track of one handclap with reflections from the soil, 10 first steps, and steps after the plateau

From the time scale it can be deduced that the first 9 step reflections came within a total time span of 24.337 ms. This implies an average delay time of $\tau$ = 2.704 ms. Alternatively, a step width of 17 inches was reported by Huygens [4], i.e. 17x27 = 459 mm. Compare the measurement by the present author of 46 cm on the average, in agreement with Minnaert [7]. From this a corresponding delay time is calculated as $\tau$ = 2x459/340 = 2.7 ms, in splendid agreement with the calculation from the audio track. Finally, a stationary Repetition Pitch is predicted of $f_0$ = 1000/2.7 = 370 Hz-equivalent.

## 4. A gliding Repetition Pitch at Chichen-Itza

### 4.1 An optics-based (Bragg) diffraction model

At Chichen-Itza in Mexico is a Maya ruin with a pyramid (see figure 7) named El Castillo that produces an echo, in response to a handclap, which sounds like the chirp of a Quetzal bird [8]. It was felt by David Lubman and others that the periodic structure of, particularly, the central smaller steps (91 in total as an estimate by the present author) are responsible for the chirp-like sound of the echo. In search for an explanation, Declercq et al. [2] performed sonogram analysis of the echo as recorded by Lubman [8], and applied the mono-frequent single homogeneous plane wave (Bragg) diffraction theory of Claeys and colleagues.

**Figure 7** El Castillo pyramid at Chichen-Itza in Mexico (after [8])

In figure 9, the background constitutes the sonogram as reproduced from figure 8 of Declercq et al. [2]. In their text below this figure they state that "it can already be concluded that these patterns cannot simply be the result of pure Bragg diffraction and that an extra effect must be involved." Thus, their efforts do not predict the chirp-echo satisfactorily.

### 4.2 An alternative model based on Repetition Pitch

Alternatively, a consideration à la Huygens seems a first candidate to try. Therefore, an exact calculation of time delays between the individual reflections from successive steps of the staircase has to be made. Adopting the data of pyramid dimensions, handclap and sound recording positions by Lubman from Declercq et al. [2], the drawing in figure 8 is obtained. The 91 steps of the staircase are numbered n = -7 to 84, with n = 0 being the step at ear (microphone) height. Dimensions are given in meters.

The sound path length *S(n)* from source-receiver position to an individual step *n* of the staircase is calculated with Pythagoras' theorem. Then the traveling time back and forth follows as *T(n),* with a sound velocity at the site of 343 m/s. The time interval (delay time) between successive reflections at the receiver position then results as *τ(n)* (see figure 8).

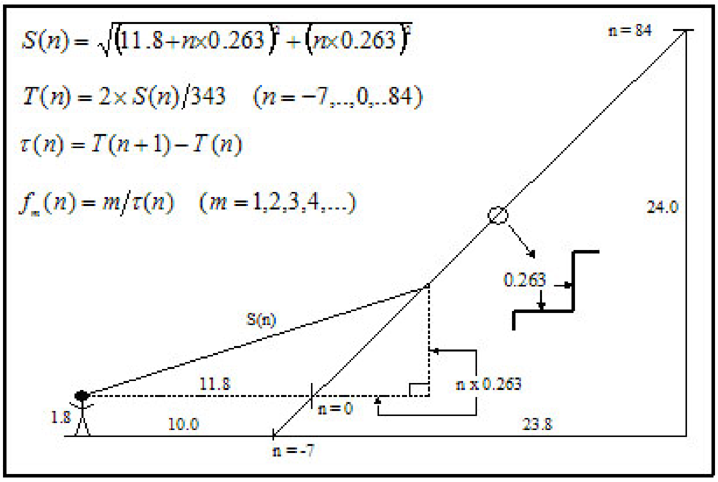


**Figure 8** Schematics of El Castillo pyramid in Chichen-Itza following the data by Lubman as adopted by Declerc et al. [2]. Dimensions are given in meters. Source-receiver position is at a height of 1.8 m at a distance of 10 m from the foot of the pyramid. Step width is 0.263 m. Steps are numbered n = -7 to 84. For the model calculations see text.

Finally, the 'harmonics' of Repetition Pitch from two successive reflections are given by $f_m(n) = m/\tau(n)$ with $m$ the harmonic number ($m$ = 1, 2, 3, 4, ….). These are indicated in figure 9 by the dotted lines, running from 0 to 177 ms (being the sum of all individual delays).

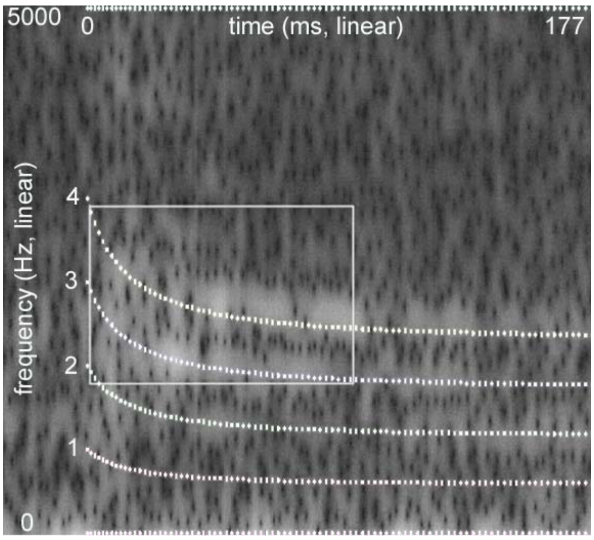


**Figure 9** Sonogram of the chirp echo in gray scale with white indicating high intensity, as a result of time-limited fourier transformation of the isolated chirp echo with a gaussian window of 2 ms, after Declercq et al.[2]. Vertical axis: frequency linear from 0 to 5000 Hz. Horizontal axis: time linear from 0 to 200 ms (N.B. The white rectangle has no specific meaning for the present consideration). Dotted lines marked 1 through 4 constitute the present model predictions following figure 8.

Notice that the dotted lines were superimposed such on the chirp sonogram that the right border of the model (177 ms point) coincided with the right border of the sonogram (200 ms point). Generally, the dotted lines fit nicely to the white regions in the sonogram. The less dark regions above 0 Hz are probably due to low frequency noise coming from the interaction of wind with the microphone.

In order to rightly appreciate the fit, one has to take into account that the 2-ms gaussian time window of the running fourier transform covered mainly 2 reflections with an average delay time interval $\tau$ of 1.95 ms (= 177/91). This means that, looking vertically in the sonogram, sinusoidal power spectra (like in the upper panel of figure 1) are represented. Further, it can be noticed that no spectral energy is present in the signal at the 'fundamental'. Nevertheless, a Repetition Pitch is correctly predicted due to the presence of the $2^{nd}$ and, more importantly, the $3^{rd}$ and $4^{th}$ harmonic (dominant according to [1]). It glides from 796 to 471 Hz-equivalent within a time span of 177 ms.

Of course, the latter values have to be stated with some reservation, because we have not at all considered the short-term characteristics of the cochlea as well as the integration characteristics of the human pitch processor. However, by informal listening to a synthesized RP-chirp following the above model, great similarity with the chirp recorded by Lubman [8] can be observed. Additionally, in this context, it is tempting to predict that a setting with continuous fountain noise would not work as good at El Castillo as in Chantilly because of the simultaneous mixing of all the different RP's in the chirp.

## Conclusions

- Huygens' (1693) observations with fountain noise at the staircase in Chantilly were confirmed in 1993 by handclapping. The latter shows a regular reflection pattern due to the steps of the staircase, resulting in a stationary Repetition Pitch of 370 Hz-equivalent. Both fountain noise and handclap are understood to be appropriate source signals.
- Contrary to (Bragg) diffraction theory, at the El Castillo step pyramid, a consideration à la Huygens, together with the properties of the human pitch processor, predict a Repetition Pitch gliding from 796 to 471 Hz-equivalent. Contrary to Chantilly, (continuous) fountain noise would probably be not a good source signal at El Castillo pyramid.

### Acknowledgements

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### References

1. Bilsen, F.A. and Ritsma, R.J., 1969/70. Repetition Pitch and its implication for hearing theory, Acustica **22**, 63-73.
2. Declercq, N.F., Degrieck, J., Briers, R. and Leroy, O., 2004. A theoretical study of special acoustic effects caused by the staircase of the EL Castillo pyramid at the Maya ruins of Chichen-Itza in Mexico, J. Acoust. Soc. Am. **116**, 3328-3335.
3. Bilsen, F.A., 1977. Pitch of noise signals: Evidence for a "central spectrum", J.Acoust. Soc. Am. **61**, 150-161.
4. Huygens, C., 1693. En envoiant le probleme d'Alhazen en France, Oeuvres Complètes **10**, 570-571.
5. Hartmann, W.M., 1997. *Signals, Sound, and Sensation*, AIP press, N.York, pp 361-376.
6. Bilsen, F.A., ten Kate, J.H., Buunen, T.J.F., and Raatgever, J., 1975. Responses of single units in the cochlear nucleus of the cat to cosine noise, J. Acoust. Soc. Am. **58**, 858-866.
7. Minnaert, M., 1970. *De Natuurkunde van het Vrije Veld*, Thieme, Zutphen, pp 45-51.
8. Lubman, D.; http://www.ocasa.org/MayanPyramid.htm, site visited on 24-11-2005.