The Tippe Top

In classical mechanics, the movement of rigid bodies is generally described by two analogous vector equations: **F** = d**p**/dt for translation of the centre of mass, and **M** = d**L**/dt for rotation around the centre of mass, with **F** the total external force, **p** the momentum, **M** the total moment of external forces, and **L** the angular momentum.

We consider the intriguing movement of the tippe top. It consists of a spherical body and a cylindrical stem, with the centre of mass CM displaced from the centre c of the sphere (see Fig. 1). When initially put into rotation around its axis of symmetry $\mathbf{e_3}$ vertical, the stem gradually moves downwards and finally the top flips over into a stable vertical rotation on the stem. Apparently the rotation has changed sign, while vector **L** has preserved its original vertical position. Further, CM has moved upwards at the cost of a decrease in magnitude of **L**. This unexpected behaviour is explained by the action of a friction force **F** at the (slipping) contact point of the top with the surface (red star pointing towards the reader).

F causes a moment **M**, which can be imagined to have vector components $\mathbf{M}_{1,2}$ and $\mathbf{M}_3$, the latter along the axis of symmetry $\mathbf{e_3}$. Likewise, the angular momentum **L** has components $\mathbf{L}_{1,2}$ and $\mathbf{L}_3$. In the beginning, $\mathbf{L}_3 = \mathbf{L}$ and $\mathbf{L}_{1,2} = \mathbf{0}$. Then, due to instability, **F** originates and the resulting $\mathbf{M}_3$ tends to decrease $\mathbf{L}_3$, while $\mathbf{M}_{1,2}$ starts to increase $\mathbf{L}_{1,2}$. As **L** remains **const**, the angle θ of the top's inclination will grow to fulfil proper vector addition. When $\theta = \pi/2$, $\mathbf{L}_3 = \mathbf{0}$ and $\mathbf{L}_{1,2} = \mathbf{L}$.

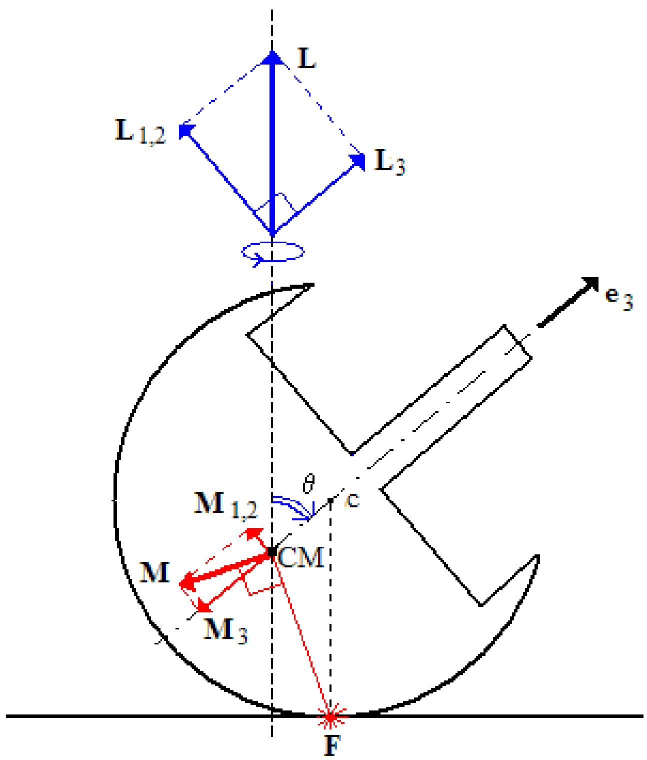

Fig. 1

Then the rotation along $\mathbf{e_3}$ changes sign and, again through the action of $\mathbf{M}_{1,2}$ and $\mathbf{M}_3$, $\mathbf{L}_3$ starts to grow at the cost of $\mathbf{L}_{1,2}$. Finally, the stem will scrape the surface (see Fig. 2) and through the action of a new frictional force **F'** with moment **M'** the top will lift itself up and strive towards a stable, though extinguishing, rotation on the stem. In fact, the component $\mathbf{L}_{1,2}$ is extinguished by the new $\mathbf{M}_{1,2}$ and $\mathbf{L}_3$ finally becomes equal to $\mathbf{L}$.

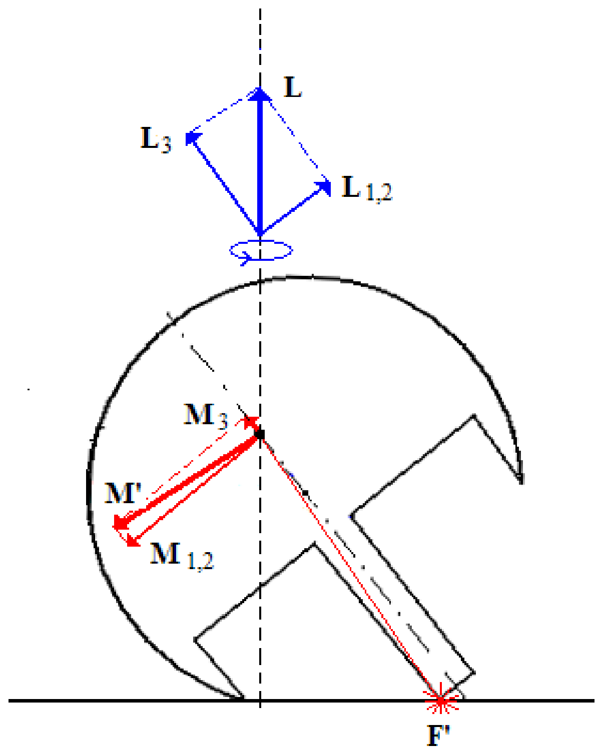

Fig. 2

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